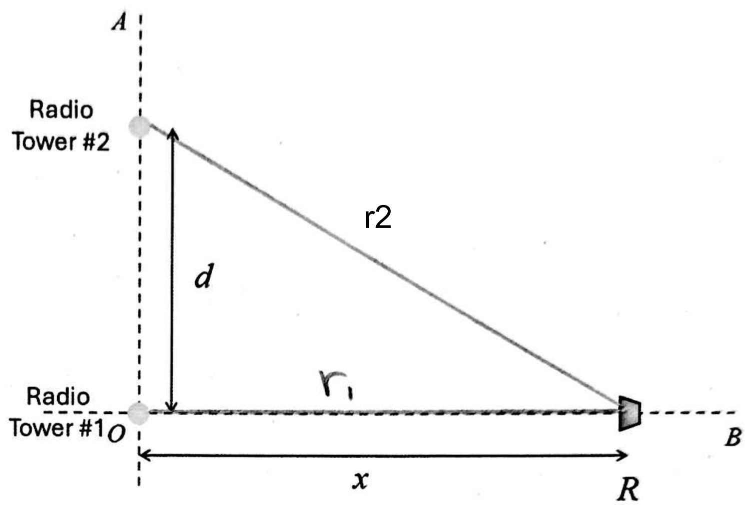
Group Members:

1. Two radio towers radiating in phase at a frequency of $5.80MHz$ are separated by a distance d as shown. A radio receiver R is located at a distance x along a line OB perpendicular to the line OA connecting the two radio towers.



The distance to the receiver x is not large as compared to the towers' separation d.

a. What is the distance r_1 between Radio Tower #1 to the receiver R?

$r_1 = x$ see above diagram

b. What is the distance r_2 between Radio Tower #2 to the receiver R?

$r_2 = (x^2+d^2)^{1/2}$ see above diagram

c. What is the path difference $\Delta = r_2 - r_1$ between the two radio sources with respect to the receiver?

$\Delta = r_2 - r_1 = (x^2+d^2)^{1/2} - x$

d. As the receiver moves closer and farther along the line *OB*, the radio signal from the two sources will get stronger and weaker due to interference. Write down the condition for constructive and destructive inference for the two radio waves.

Constructive interference:

$$\Delta = m\lambda \quad , m = 0, \pm 1, \pm 2, \ldots$$

So, we have $(x^2 + d^2)^{1/2} - x = m\lambda$, $m = 0, \pm 1, \pm 2, \ldots$

Destructive interference:

$$\Delta = (m + \tfrac{1}{2})\lambda \quad , m = 0, \pm 1, \pm 2, \ldots$$

So, we have

(*) $(x^2 + d^2)^{1/2} - x = (m + \tfrac{1}{2})\lambda$, $m = 0, \pm 1, \pm 2, \ldots$

e. As the receiver moves out (to the right) from *O*, at what distance *x* where you will find the first, second, and third destructive minimum? The distance d between tower #1 & #2 is 200. m.

$d = 200\,m$ $\quad f\lambda = c \Rightarrow \quad \lambda = \frac{c}{f} = \frac{3.00 \times 10^8\,m/s}{5.80 \times 10^6\,Hz}$

$= 51.724\,m$

Simplifying expression (*) in part d, we have

$$\cancel{x^2} + d^2 = (x + (m + \tfrac{1}{2})\lambda)^2 = \cancel{x^2} + (m + \tfrac{1}{2})^2\lambda^2 + 2x(m + \tfrac{1}{2})\lambda$$

$$2x(m + \tfrac{1}{2})\lambda = d^2 - (m + \tfrac{1}{2})^2\lambda^2$$

$$x = \frac{d^2 - (m + \tfrac{1}{2})^2\lambda^2}{2(m + \tfrac{1}{2})\lambda} = \frac{d^2}{(2m+1)\lambda} - \frac{1}{2}(m + \tfrac{1}{2})\lambda$$

First order min $(m = 0)$:

$$x = \frac{(200\,m)^2}{51.724\,m} - \frac{1}{4}(51.724\,m) = 760.\,m$$

second order min ($m=1$):

$$x = \frac{(200\,m)^2}{3(51.724\,m)} - \frac{3}{4}(51.724\,m)$$

$$= 219\,m$$

third order min ($m=2$):

$$x = \frac{(200\,m)^2}{5(51.724\,m)} - \frac{5}{4}(51.724\,m)$$

$$= 90.0\,m$$

f. Where is the last observable (largest m) destructive interference?

x must be a positive value, so

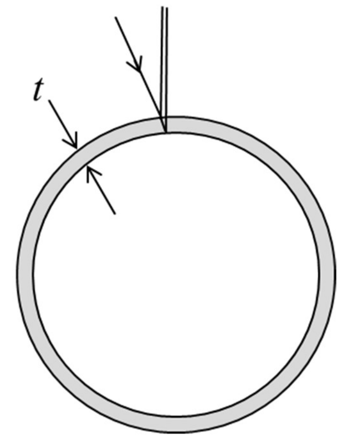
$$\frac{d^2}{(2m+1)\lambda} - \frac{1}{2}(m+\tfrac{1}{2})\lambda > 0$$

$$\frac{d^2}{\lambda^2} > (m+\tfrac{1}{2})^2$$

$$m < \frac{d}{\lambda} - \frac{1}{2} = \frac{200\,m}{51.724\,m} - \frac{1}{2} = 3.367$$

For m being integers, the largest m (integer) = 3.

2. A soap bubble with a thickness $t = 350.nm$ is show at right. Assume air to be inside and outside of the bubble and consider nearly normal incidence light rays. Take the index of refraction of the soap film to be 1.50.



   a. For an incidence light on the thin soap bubble, it will reflect back to your eyes from both the top and bottom interfaces of the soap film as indicated. Will the light reflecting back from the top of the soap film suffer a phase reversal upon reflection?

   b. Will the light reflecting back from the bottom interface of the soap film suffer a phase reversal upon reflection?

   c. Assuming almost normal incidence, write down the condition for constructive interference between the two reflected light waves.

d. Which wavelengths of light within the visible range ($390nm$ - $750nm$ 390) are strongly reflected (constructive interference) by this soap bubble (calculate all possible values within the given range)?

Let *t* be the thickness of the soap film, the condition for constructive interference in this case is,

$$2t = (m+\frac{1}{2})\lambda_n \quad or \quad 2n_{soap}t = (m+\frac{1}{2})\lambda, \quad m = 0,1,2,\cdots$$

Note that the extra path difference occurs in the optical coating so that the relevant wavelength is $\lambda_n = \dfrac{\lambda}{n_{soap}}$

This give
$$\lambda = \frac{2n_{soap}t}{\left(m+\frac{1}{2}\right)}, \quad m = 0,1,2,\cdots$$

Now, let see what wavelengths of light will be reflected constructively:

$$m = 0: \quad \lambda = \frac{2n_{soap}t}{(1/2)} = 4(1.5)(350nm) = 2100nm$$

$$m = 1: \quad \lambda = \frac{2n_{soap}t}{(3/2)} = \frac{4}{3}(1.5)(350nm) = 700nm$$

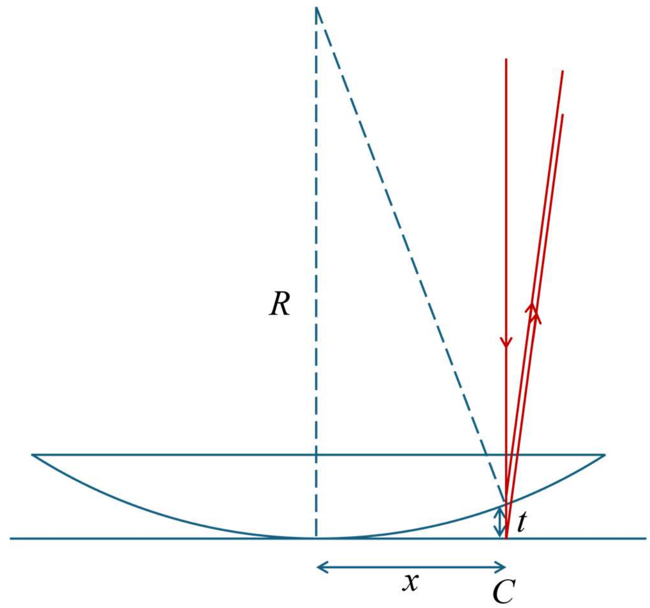
$$m = 2: \quad \lambda = \frac{2n_{soap}t}{(5/2)} = \frac{4}{5}(1.5)(350nm) = 420nm$$

$$m = 3: \quad \lambda = \frac{2n_{soap}t}{(7/2)} = \frac{4}{7}(1.5)(350nm) = 300nm$$

3. The first and the last wavelengths are outside of the visible range so that the ONLY answers should be:

$$\lambda = 700.\,nm \quad (red)$$
$$\lambda = 420.\,nm \quad (violet)$$

3. Newton's rings are visible when a planoconvex lens is places on a flat glass surface. For a particular lens with an index of refraction of $n_{lens} = 1.50$ and a glass plate with an index of $n_{plate} = 1.80$, the m-th bright ring (m^{th} order constructive inference) is observed at a distance of $x = 0.650mm$ from the center.

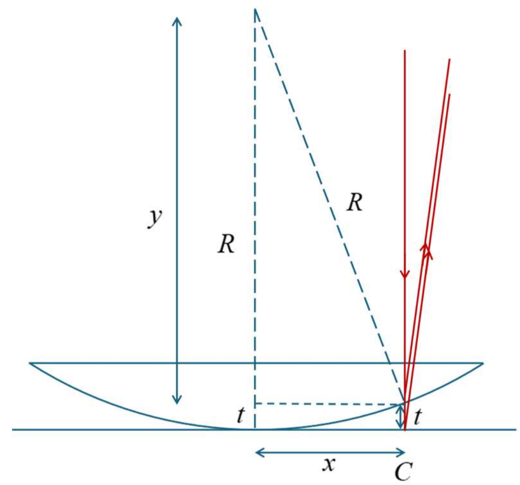


   a. For a bright ring to be seen at *C*, write down the condition for a *m*-th order constructive interference in terms of the thickness of the air-gap between the lens and the glass *t* and the wavelength λ of the light.

      Since $n_{lens}(1.50) > n_{air}(1)$, the light ray reflecting off the top interface of the air-gap will NOT suffer a phase reversal upon reflection but since $n_{air}(1) < n_{plate}(1.80)$, the light ray reflecting off the bottom interface of the air-gap will suffer a phase reversal.

      So, for consideration of constructive interference for the two rays reflecting off the top and bottom interface of the air-gap, one must take an additional half of a wavelength λ into account and this gives the condition, $2t = \left(m + \frac{1}{2}\right)\lambda$.

b. Assuming the thickness t of the air gap is small as compared to R and x and the reflected rays are nearly normal incident, express t in terms of the other geometric variables R and x.



Considering the right triangle formed by the dotted lines, $y = R - t$ and $x^2 + y^2 = R^2 \quad or \quad y^2 = R^2 - x^2$. So, we can write down the following equation,

$R - t = \sqrt{R^2 - x^2}$ or,

$t = R - \sqrt{R^2 - x^2}$

c. Assuming the curvature of the lens R is much larger than the wavelength λ of the incidence light, show that the location to a bright right x is proportional to $\sqrt{\lambda R}$.

Now, putting in the constructive inference into the above equation, we have,

$$R - \sqrt{R^2 - x^2} = \frac{1}{2}\left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \cdots$$

Moving terms around and squaring the equation gives,

$$\sqrt{R^2 - x^2} = R - \frac{1}{2}\left(m + \frac{1}{2}\right)\lambda$$

$$\cancel{R^2} - x^2 = \cancel{R^2} - \left(m + \frac{1}{2}\right)\lambda R + \frac{1}{4}\left(m + \frac{1}{2}\right)^2 \lambda^2$$

Since $\lambda \ll R$, we will also have $\lambda^2 \ll \lambda R$. So, for approximation, we can drop the λ^2 term and keep the λR term only on the right side of the equation.

Then, this gives the condition for the location x of the bright rings (constructive interference),

$$x^2 = \left(m + \frac{1}{2}\right)\lambda R$$

Taking the square root, we have an equation for the location of the bright rings in terms of $\sqrt{\lambda R}$,

$$x = \sqrt{\left(m + \frac{1}{2}\right)}\sqrt{\lambda R}, \quad m = 0, \pm 1, \pm 2, \cdots$$